4B17 Week 4 Tutorial - Kinematic Differential Equations

Q1. (From 2014-2015 exam) A male acrobat is executing a complex sideways somersault manoeuvre.

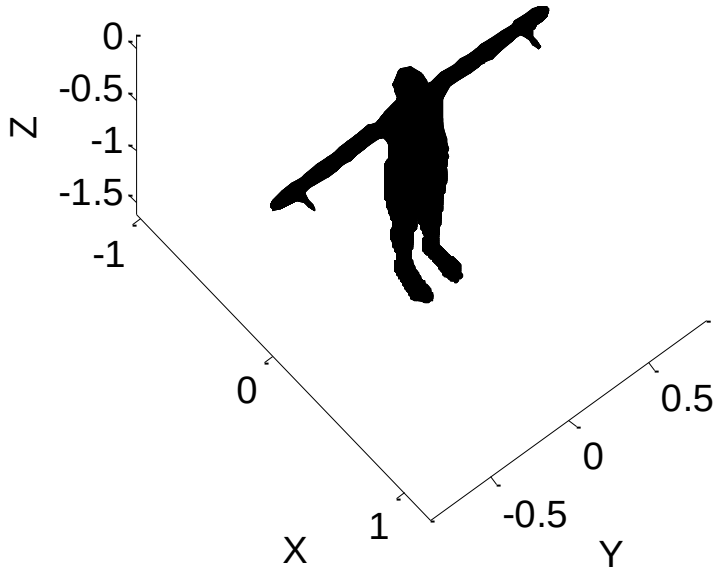


Figure Q2.

In the diagram above, he is shown at the instant of leaving the ground, initially aligned with the global XYZ coordinate system, and with the following angular velocity components from thrusting his feet against the ground:

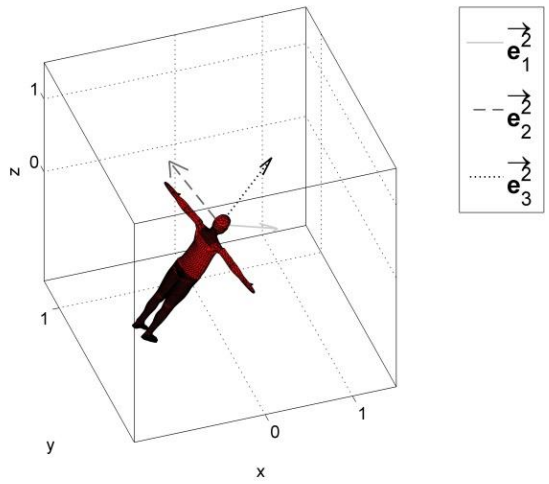
$$\{\vec{\boldsymbol{\omega}}\}^{XYZ} = \begin{Bmatrix} 0.1 \\ 4 \\ 0.2 \end{Bmatrix} rad/s.$$

Estimate the rotation matrix which defines his orientation after 10ms. You should use a time step of 5ms and can assume the angular velocity remains unchanged.

Answer:

$$[\mathrm{A}^{21}] = \begin{bmatrix} 0.996 & 0.002 & -0.04 \\ -0.002 & 1 & 0.001 \\ 0.04 & -0.001 & 0.9996 \end{bmatrix}.$$

Q2. (From 2017-2018 exam) The Figure shows a skydiver at the instant when she is executing a move in which the whole body is rotated with respect to the global coordinate system.



A fixed base $\underline{\boldsymbol{e}}^1$ has unit vectors resolved in the global coordinate system (XYZ):

$$\{\vec{\boldsymbol{e}}_1^1\}^{XYZ} = \begin{Bmatrix}1\\0\\0\end{Bmatrix}, \qquad \{\vec{\boldsymbol{e}}_2^1\}^{XYZ} = \begin{Bmatrix}0\\1\\0\end{Bmatrix}, \qquad \{\vec{\boldsymbol{e}}_3^1\}^{XYZ} = \begin{Bmatrix}0\\0\\1\end{Bmatrix}.$$

A rotated base $\underline{\boldsymbol{e}}^2$ is fixed in the natural coordinate system of the head as shown. The rotation matrix for $\underline{\boldsymbol{e}}^2 = [A^{21}]\underline{\boldsymbol{e}}^1$ is:

$$[A^{21}] = \begin{bmatrix} 0.8847 & 0.2557 & -0.3898 \\ -0.2060 & 0.9645 & 0.1651 \\ 0.4182 & -0.0658 & 0.9060 \end{bmatrix}.$$

At the instant shown, the whole body has the following global angular velocity components: $\{\vec{\boldsymbol{\omega}}\}^{XYZ} = \begin{Bmatrix}1\\7\\2\end{Bmatrix} rad/s.$

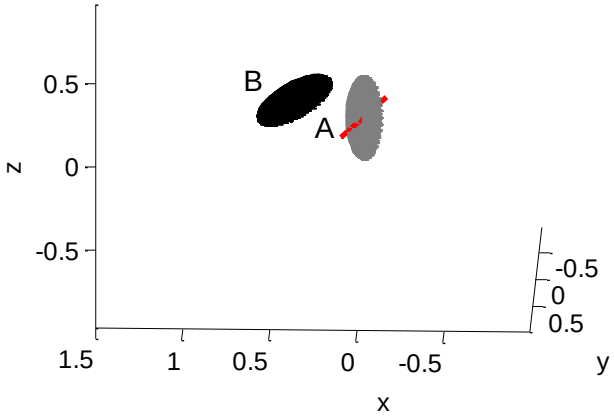
Estimate the rotation matrix defining her whole body orientation after 5ms assuming her angular velocity vector remains constant in that time.

Answer:

$$[A^{21}] = \begin{bmatrix} 0.8685 & 0.2665 & -0.4195 \\ -0.2099 & 0.9616 & 0.1771 \\ 0.4506 & -0.0661 & 0.8910 \end{bmatrix}.$$

Q3. (From 2017 exam) A rugby ball is kicked by a placekicker and is then in free flight, see Figure. The long axis of the rugby ball is initially aligned with the global Z direction (A), such that a coordinate system fixed in the ball is initially aligned with the global XYZ coordinate system. You can assume that the foot to ball contact occurs instantaneously and that the subsequent angular velocity $\vec{\boldsymbol{\omega}}$ of the ball has global XYZ components:

$$\{\vec{\boldsymbol{\omega}}\}^{XYZ} = \begin{Bmatrix} 0.5 \\ -5 \\ 1 \end{Bmatrix} rad/s \ .$$



Find the rotation matrix for the ball after 10 ms (B), assuming a 5 ms time step.

Answer:

$$[A^{21}] = \begin{bmatrix} 0.993 & 0.0099 & 0.05 \\ -0.0101 & 1 & 0.0049 \\ -0.05 & -0.0051 & 0.9994 \end{bmatrix}.$$