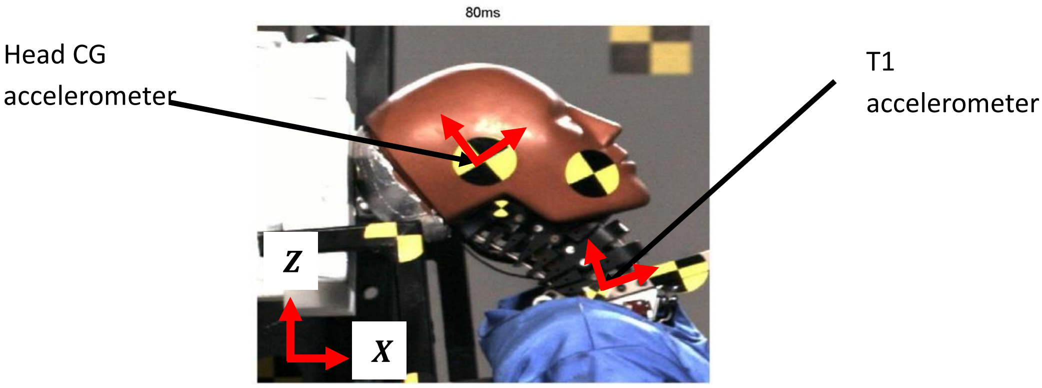
4B17 Week 3 tutorial – 3D Rotations

Q1. (from 2013-2014 Exam) During a rear impact test to assess the neck injury risk of an occupant in a wheelchair used in transit, an instrumented crash test dummy is used to measure the relative acceleration between the centre of gravity (CG) of the head and the base of the neck at the first thoracic vertebra (T1) using triaxial accelerometers fixed at these two locations in the dummy. Prior to the impact, the orientation of these accelerometers is parallel to a reference base in which the first unit vector is along the direction of vehicle travel and the third unit vector is in the vertically upward direction. After 80 ms, the crash test dummy head has moved rearward and rotated relative to the torso in the classic whiplash mode as shown.



At 80ms, the following two accelerometer readings were observed (in SI units):

$$\{\vec{\boldsymbol{a}}\}^{T1} = \begin{Bmatrix} 110 \\ -4 \\ 80 \end{Bmatrix} \quad \text{and} \quad \{\vec{\boldsymbol{a}}\}^{head\ CG} = \begin{Bmatrix} 250 \\ 5 \\ 20 \end{Bmatrix}.$$

The orientation of T1 and the head at 80ms are known from high-speed video: motion has stayed approximately in the sagittal plane (the plane shown in the image above) and T1 has rotated anticlockwise with respect to the reference base by 15 degrees while the head has rotated anticlockwise with respect to the reference base by 30 degrees as shown.

Estimate the components of the relative acceleration vector between the head CG and the first thoracic vertebra T1 at 80ms expressed in the coordinate system of the reference base.

Ans: $\vec{\boldsymbol{a}}_{head\ rel\ to\ T1} = \begin{Bmatrix} 121 \\ 9 \\ 37 \end{Bmatrix} m/s^2$

Q2. For the following rotation matrix:

A21 =

0.3584	0.8613	-0.3603
-0.6638	0.5064	0.5503
0.6564	0.0420	0.7532 …

Find the components of the screw axis of rotation and the angle of rotation around the screw axis.

Ans: $n_1 = 0.2672;\ n_2 = -0.5345;\ n_3 = 0.8018;\ \emptyset = 72deg$

Q3. For the following rotation matrix:

A21 =

0.3584	0.8613	-0.3603
-0.6638	0.5064	0.5503
0.6564	0.0420	0.7532

Find the Euler-Rodrigues parameters associated with this rotation matrix.

Ans: $q_0 = 0.8090;\ q_1 = 0.1571;\ q_2 = 0.3142; q_3 = 0.4713;$

Q4 (From 2015-2016 Exam) A fixed base $\underline{\boldsymbol{e}}^1$ has unit vectors $\vec{\boldsymbol{e}}_1^1$, $\vec{\boldsymbol{e}}_2^1$, $\vec{\boldsymbol{e}}_3^1$ resolved in the global coordinate system (XYZ):

$$\{\vec{\boldsymbol{e}}_1^1\}^{XYZ} = \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix}, \qquad \{\vec{\boldsymbol{e}}_2^1\}^{XYZ} = \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix}, \qquad \{\vec{\boldsymbol{e}}_3^1\}^{XYZ} = \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix}.$$

A rotated base $\underline{\boldsymbol{e}}^{\mathbf{2}}$ has unit vectors $\vec{\boldsymbol{e}}_1^2$, $\vec{\boldsymbol{e}}_2^2$, $\vec{\boldsymbol{e}}_3^2$ resolved in the global coordinate system (XYZ):

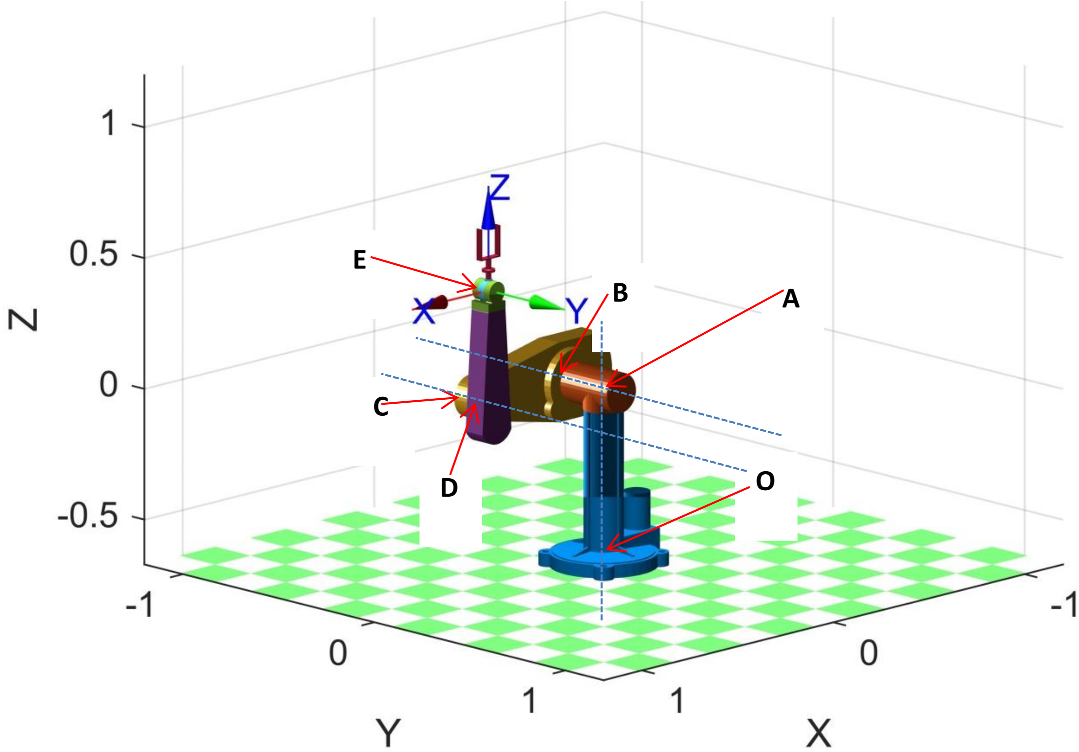
$$\{\vec{\boldsymbol{e}}_1^2\}^{XYZ} = \begin{Bmatrix} 0.7174 \\ -0.1477 \\ 0.6808 \end{Bmatrix}, \qquad \{\vec{\boldsymbol{e}}_2^2\}^{XYZ} = \begin{Bmatrix} 0.2514 \\ 0.9663 \\ -0.0553 \end{Bmatrix}, \qquad \{\vec{\boldsymbol{e}}_3^2\}^{XYZ} = \begin{Bmatrix} -0.6497 \\ 0.2108 \\ 0.7304 \end{Bmatrix}.$$

Find the rotation matrix linking base $\underline{\boldsymbol{e}}^2$ to base $\underline{\boldsymbol{e}}^1$.

Ans:

$$[\mathrm{A}^{21}] = \begin{bmatrix} 0.7174 & -0.1477 & 0.6808 \\ 0.2514 & 0.9663 & -0.0553 \\ -0.6497 & 0.2108 & 0.7304 \end{bmatrix}.$$

Q5. (From 2017-2018 exam) A 3-axis robot is shown. The robot configuration is defined by rotation θ_1 about axis A0, rotation θ_2 about axis BA and rotation θ_3 about axis DC. The link lengths are: A0 = 0.2m; BA = 0.04m; CB = 0.1m; DC = 0.02m and ED = 0.08m. For the following calculations state and briefly explain any assumptions.



a) Find the orientation of the end-effector coordinate system at E relative to the global coordinate system when $\theta_1 = 30°;\ \theta_2 = 45°;\ \theta_3 = 0°$;

b) Find the global end-effector position relative to O for $\theta_1 = 30°;\ \theta_2 = 45°;\ \theta_3 = 0°$.

$$Ans\ (a) = \begin{bmatrix} 0.612 & 0.354 & -0.707 \\ -0.5 & 0.866 & 0 \\ 0.612 & 0.354 & 0.707 \end{bmatrix};\ (b) = \begin{Bmatrix} -0.0025 \\ -0.0673 \\ 0.3107 \end{Bmatrix} m$$